Below 590 MeV a three-parameter curve (A,B,C) fit the data sufficiently well. At 590 and 700 MeV the 5-parameter fits seemed to make a significant improvement, but the errors in D and E are still large and great significance should not be given to these parameters. The values were $D=1.6\pm0.8$, $E=0\pm1$ and $D=-0.1\pm0.9$, $E=2.8\pm1.3$, all in μb/sr, at the two energies, respectively. The results for A, B, and C as a function of energy are shown in Fig. 6. The errors given are derived from the absolute errors in the cross sections. The adopted 3-parameter fits are indicated by the solid lines in Figs. 4 and 5; for the 500-MeV data the 5-parameter fit is also shown.

In Fig. 4 are also drawn the theoretical curves of DeTollis and Verganelakis[20] and of Gourdin and Salin.[21] DeTollis and Verganelakis supplement the dispersion theory expression of Chew *et al.* with bipion and tripion contributions. They are able to fit the data at 200 and 320 MeV reasonably well but, as seen in Fig. 4, they disagree sharply with our forward-angle data at 400 MeV. On the other hand, Gourdin and Salin attempt to fit the data using an isobar model; they find no necessity for any pion-pion terms. As seen in Fig. 4 they fit the data at 400 MeV much better than DeTollis and Verganelakis. At higher energies the agreement with the data, though not perfect, is qualitatively satisfactory; a typical example of the degree of agreement is shown in Fig. 5 for 450 MeV. Finally at 800 MeV they are in severe disagreement with the data of Talman *et al.*[7]

[20] B. DeTollis and A. Verganelakis, Nuovo Cimento **22**, 406 (1961).
[21] M. Gourdin and P. Salin, Nuovo Cimento **27**, 193 (1963); P. Salin (unpublished).

ACKNOWLEDGMENTS

We wish to thank Dr. Martin Feldman for assistance of various kinds, and Cecil Chin-Fatt for help in taking the data.

PHYSICAL REVIEW VOLUME 132, NUMBER 3 1 NOVEMBER 1963

Intensity of Upward Muon Flux due to Cosmic-Ray Neutrinos Produced in the Atmosphere*

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(Received 17 June 1963)

Calculations have been performed to determine the upward going muon flux leaving the earth's surface after having been produced by cosmic-ray neutrinos in the crust. Only neutrinos produced in the earth's atmosphere are considered. Rates of the order of one per 100 sq m/day might be expected if an intermediate boson exists and has a mass less than 2 BeV.

THE possibility of detecting high-energy neutrinos in cosmic rays has been discussed by many authors.[1–3] In particular, it has been pointed out[2] that an effective method is to use the earth's crust as a target and observe high-energy muons which are produced by cosmic-ray neutrinos coming from the opposite side of the earth. Such an experiment, if feasible, may well be the only means to yield direct information on very high-energy neutrino reactions. Because of the rapid decrease of cosmic-ray neutrino flux with increasing energy, the number of muons produced in this way depends sensitively on the high-energy behavior of neutrino reactions. In this letter, the expected upward muon flux at the surface of earth is calculated in some detail under certain assumptions on the nature of the weak interactions.

We first consider the case that weak interactions are transmitted by intermediate bosons, called $W^\pm$, of a mass m_W which is not much bigger than 1 BeV. Since we are only interested in very high-energy neutrinos (from about 10 to several hundred BeV) the dominant reactions are the coherent processes

$$\nu_\mu+Z\rightarrow\mu^-+W^++Z \quad (1)$$

* Work supported in part by the United States Atomic Energy Commission.
† Alfred P. Sloan Fellow.
[1] K. Greisen, in *Proceedings of International Conference for Instrumentation in High Energy Physics* (Interscience Publishers, Inc., New York, 1960), p. 209.
[2] G. T. Zatsepin and V. A. Kuz'min, Zh. Eksperim. i Teor. Fiz. **41**, 1818 (1961) [translation: Soviet Phys.—JETP **14**, 1294 (1962)].
[3] Compare, also M. A. Markov and I. M. Zheleznykh, Nucl. Phys. **27**, 385 (1961).

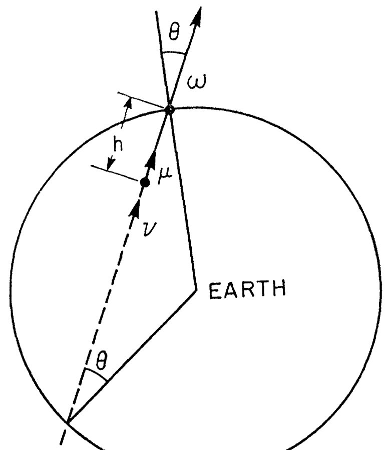

FIG. 1. Schematic drawing of muons produced by cosmic-ray neutrinos interacting with the earth's crust.

and

$$\bar{\nu}_\mu+Z\rightarrow\mu^{+}+W^{-}+Z\,, \qquad (2)$$

where Z is the target nucleus. Such a reaction may take place at a vertical depth, say, $(h\cos\theta)$ from the surface of the earth and produce a muon of energy μ moving at an angle θ with respect to the upward vertical direction. (See Fig. 1.) Let ω be the final muon energy after it travels through the distance h. At the surface of the earth the muon flux within the energy interval $d\omega$ and solid angle $d\Omega$ is given by

$$dM=d\omega d\Omega\int_{\omega}^{\infty}d\mu\int_{\mu+m_W}^{\infty}d\nu\, N_\nu(d\sigma/d\mu)(-d\mu/dh)^{-1}n\,, \quad (3)$$

where $(d\sigma/d\mu)$ is the differential cross section per atom for reactions (1) or (2), n is the number of atoms per unit volume, ν is the energy of the neutrino, N_ν is the neutrino flux per unit energy interval per unit solid angle, and $(-d\mu/dh)$ is the ionization loss of muon energy per unit length.

For very high-energy muons, the energy loss is given approximately by[4] $(\hbar=c=1)$

$$-(d\mu/dh)\cong 2\pi nZ\alpha^2(m_e^{-1})\times\{\ln[(\alpha\pi nZ)^{-1}m_e^{3}(\mu/m_\mu)^2]-1\}\,, \quad (4)$$

where Z is the average number of electrons per atom, α is the fine structure constant and m_e, m_μ are, respectively, the mass of electron and muon. For all subsequent calculation we use silicon as the relevant atom, since almost all the neutrino reactions are taking place in the earth's crust.

The neutrino flux that is produced in the atmosphere can be decomposed into three terms[1,2]:

$$N_\nu=N_\nu(\mu)+N_\nu(\pi)+N_\nu(K)\,, \qquad (5)$$

due to the decays of μ, π, and K, respectively.

By using the observed muon spectrum[5] in cosmic rays, it is straightforward to derive the total number $D(E_\mu,\cos\theta)dE_\mu d\Omega$ of μ decays for muons of energy E_μ which decay in the atmosphere, per unit area per unit time. The corresponding neutrino flux is given by

$$N_\nu(\mu)\cong\frac{1}{3}\int_{\nu}^{\infty}D(E_\mu,\cos\theta)\times[5-9(\nu/E_\mu)^2+4(\nu/E_\mu)^3]\left(\frac{dE_\mu}{E_\mu}\right). \quad (6)$$

In (6) we include only the μ-neutrinos from μ decays, since e neutrinos can not directly produce $\mu^{\pm}$.

Using the results of Greisen,[1] the neutrino flux due to π decays is

$$N_\nu(\pi)\cong\frac{7}{3}\int_{(7/3)\nu}^{\infty}f(E_\pi)\left[\frac{B_\pi}{B_\pi+E_\pi\cos\theta}\right]\left(\frac{dE_\pi}{E_\pi}\right), \quad (7)$$

where $f(E)=0.16(E/\text{BeV})^{-2.6}$ per BeV-cm^2-sr and $B_\pi\cong 120$ BeV.

To obtain the neutrino flux due to K decays, we assume the K-meson spectrum has approximately the same energy dependence as the π-meson spectrum. Therefore, $N_\nu(K)$ is given by

$$N_\nu(K)\cong(br)\int_{\nu}^{\infty}f(E_K)\left[\frac{B_K}{B_K+E_K\cos\theta}\right]\left(\frac{dE_K}{E_K}\right), \quad (9)$$

where $B_K\cong 900$ BeV, f is given by (8), r is the average production ratio of (K/π) at the same energy in a high-energy nucleon-nucleon collision, and b is the branching ratio of $K_{\mu 2}$ decay. For definiteness, we take $(br)=10\%$.

Because of the much greater fractional energy given to the neutrino in a $K_{\mu 2}$ decay as compared to that in a $\pi_{\mu 2}$ decay, the effect of $N_\nu(K)$ is especially important for high-energy neutrinos. In contrast, the observed muon spectrum is hardly influenced at all by the presence of $K_{\mu 2}$ decay, provided (br) is small.

The complete expressions of the differential cross sections for reactions (1) and (2) have been given in the literature.[6] Let $F(q)$ be the charge form factor of the target nucleus:

$$F(q)=[1+\tfrac{1}{12}q^2R^2]^{-2}\,, \qquad (10)$$

where q is the 4-momentum transfer of the nucleus and R is its charge radius. For the detection of cosmic-ray neutrinos, one is only interested in relativistic μ and W; furthermore, (q/μ) and (q/W) are both much smaller than unity. (μ and W refer to the energy of the muon

[4] See, for example, H. A. Bethe and J. Ashkin, in *Experimental Nuclear Physics* I, edited by E. Segrè (John Wiley & Sons, Inc., New York, 1952), p. 166.

[5] J. Pine, R. J. Davisson, and K. Greisen, in *Proceedings on the Moscow Conference on Cosmic Rays* edited by N. M. Gerafimova, and A. I. Nikifhov (Moscow, 1960), Vol. 1, p. 295.

[6] T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Letters **7**, 429 (1961). An asymptotic expression of the coherent cross section has been previously derived for the extremely high-energy region: $\ln[2(12)^{1/2}\nu/m_W^2R]\gg 1$ which, unfortunately, is not applicable even in the present case.

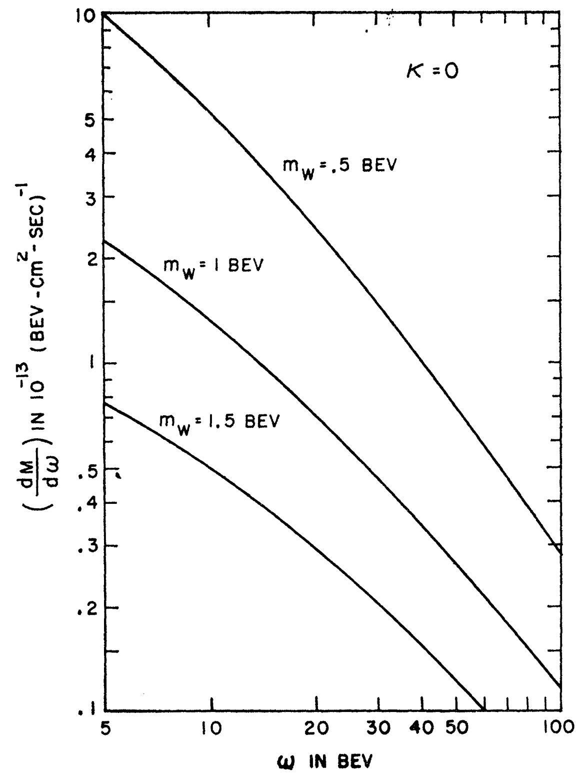

FIG. 2. Flux of muons as a function of energy for $\kappa=0$ and different m_W values.

and the intermediate boson in the laboratory system.) It can be shown that in such a case the differential cross section for reaction (1) or (2) with the production of a W of helicity s is given by

$$d\sigma_s=Z^2\alpha^2(\pi\sqrt{2})^{-1}G(d\mu/\nu)\sum_{m=0}^{3}C_s{}^mA_m\,, \tag{11}$$

where

$$A_m(x)=\int_x^\infty y^{-3+m}B(y)dy\,, \tag{12}$$

$$B(y)=\int_y^\infty (1+z^2)^{-4}(z^2-y^2)z^{-3}dz$$
$$=\tfrac{1}{2}\{(1+4y^2)\ln[y^{-2}(1+y^2)]-4+(1+y^2)^{-1}+\tfrac{1}{6}(1+y^2)^{-2}\}\,, \tag{13}$$

$$x=(\Delta/W)+(\delta/\mu)\,,$$
$$\Delta=2^{-1}(12)^{-1/2}m_W{}^2R\,, \tag{14}$$
$$\delta=2^{-1}(12)^{-1/2}m_\mu{}^2R\,,$$

G is the Fermi coupling constant and the subscripts $s=R$, L, l denote, respectively, a right-handed $W^\pm$, a left-handed $W^\pm$, and a longitudinal $W^\pm$. In Table I,

TABLE I. The coefficients $C_s{}^m$ ($s=R$, L, and l). See Eqs. (14), (15), and (16) for the definitions of x, κ, d_1, and d_2.

m	$(\Delta\mu)^{-1}W^2C_R{}^m$	$(\Delta\nu^2)^{-1}W^2\mu C_L{}^m$	$\nu^{-1}WC_l{}^m$
0	0	0	$\frac{1}{2}(1-\kappa)^2[1+(\mu/\nu)^2]$
1	$5-2+\kappa^2$	$4(1-\kappa)^2(\mu/\nu)^2$	$-x\nu^{-1}WC_l{}^0+d_1$
2	$4(\kappa-3)x$	$-4[2+(1-\kappa)(\mu/\nu)]x$	$-xd_1+d_2$
3	$8x^2$	$8x^2$	$-xd_2$

we tabulate the coefficients $C_s{}^m$, where

$$d_1=-2[(2\Delta/W)+(\delta/\mu)](1-\kappa)[1+(\mu/\nu)]\,, \tag{15}$$
$$d_2=4[(2\Delta/W)+(\delta/\mu)]^2\,,$$

and κ is related to the magnetic moment of W by

$$(\text{magnetic moment})=(2m_W)^{-1}e(1+\kappa)\,. \tag{16}$$

It is important to notice that (11) is applicable for *arbitrary* value of $(m_W{}^2R/\nu)$ provided that (q/μ), (q/W) are small compared to unity and that $\mu^\pm$ and $W^\mp$ are relativistic.

Combining (3)–(11), the differential muon flux $(d^2M/d\omega d\Omega)$ and its integrated spectrum $(dM/d\omega)$ and M at the surface of earth have been calculated numerically by assigning different values of m_W and κ, for $\cos\theta$ varying from 0 to 1 and for ω ranging from 5 to 1000 BeV. Some typical examples of $(d^2M/d\omega d\Omega)$ and $(dM/d\omega)$ are given in Figs. 2 and 3. The integrated values of muon flux for $\omega\geq 5$ BeV are given in Table II. This numerical calculation was made with the IBM 7090 computer at Columbia University.

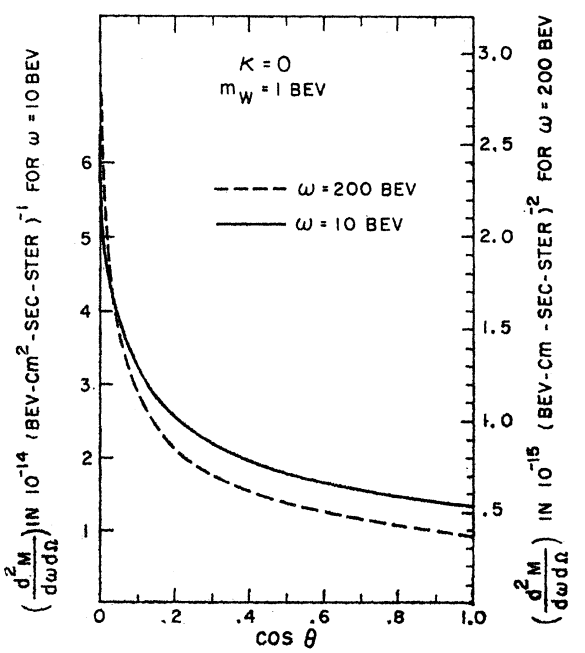

FIG. 3. Angular distributions of muons for $m_W=1$ BeV, $\kappa=0$, and $\omega_\mu=10$ and 200 BeV. The angle θ is between the momentum of muon and the vertical direction.

TABLE II. Muon flux in units of $(\text{day})^{-1}$ $(10\text{ m})^{-2}$, integrated over (2π) solid angle and muon energy from 5 BeV up. Only the muons produced by reactions (1) and (2) are included.

(m_W/BeV) \ κ	2	1	0
0.5	6.46	3.15	1.48
0.75	3.31	1.67	0.801
1	1.93	0.998	0.491
1.5	0.833	0.446	0.228
2	0.432	0.237	0.125

It should be emphasized that this calculated muon flux is only a lower limit since it does not include (i) muons produced in the $W^{\pm}$ decays; (ii) muons produced by the noncoherent W-producing processes; (iii) neutrino reactions in which $W^{\pm}$ is not produced in the final state; (iv) neutrinos not produced in the atmosphere. The last effect has been discussed by Greisen.[1]

The effect of (i) is important only if the leptonic decays of $W^{\pm}$ turn out to be the dominant modes. In such a case an increase of muon flux $\lesssim 25\%$ from the above calculation may be expected.

The effect of (ii) is important only for relatively low-energy neutrinos. At very high energy, the cross section for noncoherent W-producing reaction is expected to be about Z^{-1} of that of the coherent reactions. The effect of (iii) is due to reactions such as:

$$\nu_\mu \text{ (or } \bar{\nu}_\mu) + \text{nucleon} \rightarrow \mu^- \text{ (or } \mu^+) + \text{nucleon} + \text{pions} + \cdots . \quad (17)$$

If $W^{\pm}$ does not exist or if it exists but its mass is too large, (17) is, then, the only relevant reaction. Due to the presence of strong interactions, the cross section for (17) is not known theoretically. Nevertheless, it is expected to be proportional to $G^2\langle q^2\rangle$ where $\langle q^2\rangle$ is the average of the square of the 4-momentum transfer given to the lepton.[7] Let $\langle N_\pi\rangle$ and $\langle p_\perp^2\rangle_\pi$ be the average multiplicity of pions and the average of the square of the *transverse* momentum of the pions. By assuming a random distribution of the direction of the vector $(\mathbf{p}_\perp)_\pi$, we have

$$\sigma_{\text{tot}} \sim G^2\langle q^2\rangle \sim G^2\langle N_\pi\rangle\langle p_\perp^2\rangle_\pi . \quad (18)$$

We further assume that at very high energy the behaviors of $\langle N_\pi\rangle$ and $\langle p_\perp^2\rangle_\pi$ are similar to that in a high-energy nucleon-nucleon collision; i.e.,

$$\langle p_\perp^2\rangle_\pi \simeq \text{const}$$

and

$$\langle N_\pi\rangle \simeq E_{\text{c.m.}}{}^{\beta},$$

where $E_{\text{c.m.}}$ is total energy in the center-of-mass system and β is very roughly $\approx \frac{1}{2}$. In such a case, the total cross section becomes proportional to $G^2E_{\text{c.m.}}{}^{\beta}$, instead of the usual $G^2E_{\text{c.m.}}{}^2$ expression for a neutrino interacting with particles with no strong interactions. A convenient phenomenological formula for the total cross section of (17) may, therefore, be written as (averaged over both neutron and proton)

$$\sigma_{\text{tot}} \cong (C/\pi)G^2m_p{}^2(\nu/m_p)^{\beta/2}, \quad (19)$$

where m_p is the nucleon mass, ν is the neutrino energy in the laboratory system ($\nu \gg m_p$), and C is a dimensionless proportionality constant (for a high energy ν_μ interacting with a "bare" nucleon with no strong interactions $C=1$ and $\beta=2$). The resulting muon flux can be computed by using (4) and (5).

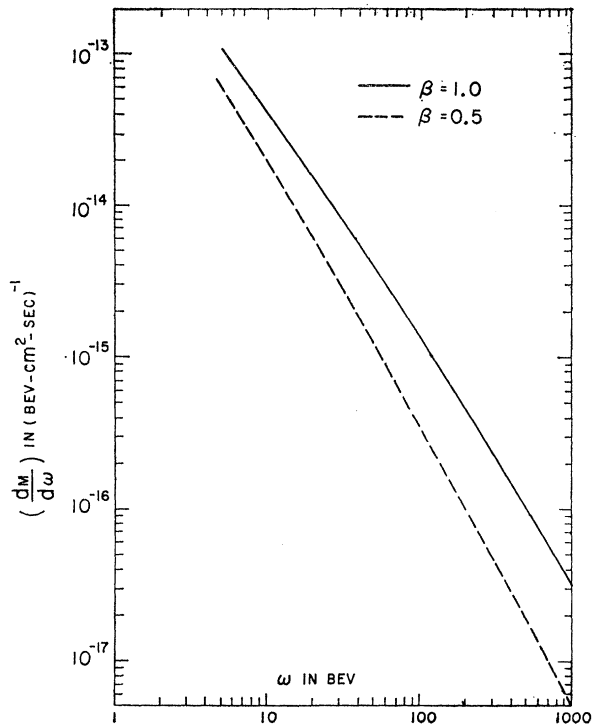

FIG. 4. Energy spectrum of muons leaving the earth's surface which have been produced as the result of reaction (17).

By carrying out these numerical calculations, we find the total muon flux for $\omega \gtrsim 5$ BeV and integrated over 2π solid angle:

$$M = C[0.038] \text{ per } (10\text{ m})^2\text{—day for } \beta = \tfrac{1}{2}$$

and

$$M = C[0.095] \text{ per } (10\text{ m})^2\text{—day for } \beta = 1.$$

The spectra for these two cases are given in Fig. 4.

Therefore, it seems that (17) becomes important only if W does not exist, or if it exists but its mass is larger than 2 BeV. For smaller values of m_W, the average $\langle q^2\rangle$ can be substantially reduced by the

$$[1 + q^2/m_W{}^2]^{-1}$$

factor in the $W^{\pm}$ propagator. Thus, the muon flux due to reaction (17) may be much smaller than the above estimation.

[7] See T. D. Lee, CERN Report, p. 61–30, 1961 (unpublished).